Calculation Policy

April 2017

Aims

The aims of the policy are to provide guidance on the steps needed when teaching the four main operations: addition, subtraction, division and multiplication; as well as provide guidance on mental maths expectations for each year group.

- To ensure consistency across the school
- To outline a consistent approach to progression
- Use assessment for learning to identify suitable next steps in calculation for groups of children
- The mathematical language used to describe each operation should be encouraged and displayed on learning walls
- Children will be taught each stage through variation of question type and context

Year 1 end of year Mental Calculations expectations

Rapid Recall and Counting	Children should be able to use the following **Mental Strategies,** as appropriate for mental calculations	Children should be able to **Calculate Mentally**
• Count from 1 and from any other number forwards and backwards in 1's 2's 5's 10's • Distinguish between ordinal and cardinal numbers • Know by heart all number bonds to 20 in 3 forms…6+8 = 14, 14-8=6, 14-6=8 • Recall doubles of all numbers to at least twenty and corresponding halves • Begin to recognise two-digit multiples of 2,5,10 • Know odd and even numbers	• Re-order numbers in a calculation e.g. 2 + 8 to 8 + 2 noticing that this does not Change the answer • Begin to bridge through 10, and later 20, when adding a single digit number • Use known number facts and place value to add or subtract pairs of single-digit numbers • Add 9 to single-digit numbers by adding 10 and then subtracting 1 • Identifying near doubles, using doubles already known i.e. 8 + 7 is 7 + 7 + 1, or 8 + 8 - 1 • Use patterns of similar calculations i.e. 9 + 1 = 10, 9 + 2 = 11, 9 + 3 = 12 • Begin to partition to add numbers close to a multiple of 10 e.g. 5 + 9 = 5 + 10 – 1 (using a number line) • Bridging through numbers other than 10, e.g. 1 week = 7 days 'It is half past seven. What time was it 3 hours ago?'	• Add or subtract a single digit to or from a single digit without crossing 10 i.e. 4 + 5, 8 – 3 • Add or subtract a single digit to or from 10 • Add or subtract a single-digit to or from a 'teens' number, without crossing 20 or 10, e.g. 13 + 5, 17 – 3 • Doubles of all numbers to 20, e.g. 7 + 7, double 9 • Add or subtract 10 to any 2 digit number i.e. 32 + 10 = 42, recognising patterns and the digit that changes

Year 2 end of year Mental Calculations expectations

Rapid Recall and Counting	Children should be able to use the following **Mental Strategies,** as appropriate for mental calculations	Children should be able to **Calculate Mentally**

• Count to and beyond 100 from any number
• Count in steps of 2,3,5 and 10 from any given number
• Recognise odd and even numbers
• Count in fractions up to 10, using ½ and 2/4 equivalence
• Know by heart all number bonds that total 20
• Know by heart all addition and subtraction facts for each number up to 20
• Know by heart doubles of all number to 20
• Know by heart all halves of numbers to 20
• Recall halves of even numbers to 100
• Know by heart all multiplication facts for 2, 5 and 10 tables
• Know division facts for multiples of 2,5,10
• Know by heart all bonds of multiples of 10 up to 100 e.g. 30 + 70
	• Find 10 more and less than numbers to 100
	• Find a difference by calculating from the smaller to the larger number.
	• Reorder numbers in a calculation.
	  ○ E.g. 2 + 36 = 36 +2, 5 + 7 + 5 = 5 + 5 + 7
	• Add three small numbers by putting the largest number first and/or finding a pair totalling 10. i.e. 2 + 6 + 9 becomes 9 + 6 + 2
	• 8 + 3 + 2 becomes (8 + 2) + 3 =
	• Respond to questions such as 'Tell me three numbers that add to 20?'
	• Work out 1 + □ + 5 = 17
	• Bridging through numbers other than 10.
	• e.g. 1 year = 12 months. 10.30 to 10.45
	• Partition additions into 10s and units and then recombine mentally with 2 digit numbers that total less than 100 i.e. 24 + 12 =
	  ▪ 20 + 10 + 4 + 2 =
	  ▪ 30 + 6 = 36
	• Partitioning bridging through multiples of 10
	  ○ 6 + 7 = 6 + 4 + 3
	• 23 - 9 = 23 – 3 - 6
	• Use known number facts and place value to add or subtract pairs of numbers
	• Understanding place value to multiply and divide by 10, moving digits to right to multiply by 10 and to the left to divide by 10
	• i.e. 17 x 10 = 170, 30 ÷ 10 = 3.
	• Add or subtract 9, 19 ,11 or 21 by rounding and compensating.
	i.e. 37 +9 = 37 + 10 – 1 = 46
	• Using near doubles e.g. 13 + 14 is double 14 and subtract 1 or double 13 and add 1. understanding halving as the inverse of doubling
	• Use the relationship between addition and subtraction i.e. 8 + 7 = 15 therefore 15 – 8 = 7 and 15 – 7 = 8
	• Use knowledge of number facts and place value to multiply and divide by 2, 5, and 10
	• Uses patterns of similar calculations. e.g. 12 + 7 = 19 , 120 + 70 = 190
		• Find one more / less than a number
		• Find 10 more / less than a number
		• Add or subtract a single-digit to or from any two-digit without crossing the tens boundary e.g. 62 + 4, 38 - 7
		• Add or subtract a single-digit to or from 10
		e.g. 60 + 5, 80 - 7
		• Add or subtract any teens number to any two-digit number, without crossing the tens boundary e.g. 24 + 14, 48 - 13
		• Find what must be added to any two-digit multiple of 10 to make 100
		e.g. 70 + □ = 100
		• Add or subtract a multiple of 10 to or from any two-digit number when the difference is less than 10 e.g. 78 – 71 or 52 - 48
		• Doubles of all numbers to at least 20. Double any multiple of 5 up to at least 50
		e.g. double 35.
		• Halve any multiple of 10 up to 100
		e.g. halve 50
		Mental addition and subtraction of two, two digit numbers, totalling less than 100

Year 3 end of year Mental Calculations expectations

Rapid Recall and Counting	Children should be able to use the following **Mental Strategies,** as appropriate for mental calculations	Children should be able to **Calculate Mentally**
• Order numbers and compare to 1000 • Count in multiples of 2,3,4,5,8 and 10 forwards and backwards. • Count in 50's and 100's • Know 10 or 100 more than numbers to 1000 • Count in fractions to 10, also count forwards and backwards in tenths. Recognise equivalents • Know by heart addition and subtraction facts for each number up to 20 • Know by heart all sums and differences of multiples of 10 up to 100 • Know by heart all doubles of multiples of 5 and 10 up to 100 • Know by heart all halves of multiples of 10 up to 100 • Know by heart all multiplication facts for2, 3, 4, 5, ,8,10 up to 12 • Know the corresponding division facts for above tables to 12 • Recognise multiples of 2, 5, 10 up to 1000 • Know all pairs of multiples of 100 with a total of 1000 e.g. 800 + 200 • Know all pairs of multiples of 5 with a total of 100 i.e. 56 + 35	• Find a difference by calculating from the smaller to the larger number, e.g. 82 – 47 = 15 • Reorder numbers in a calculation e.g. 12-7-2 = 12-2-5 • Add three or four small numbers by putting the largest number first and/or finding pairs totalling 10 • Partition into tens and units and recombine For calculating, answers to exceed 100 • Bridge through a multiple of ten and adjust be able to do this with 3 digit numbers e.g. 149 + 32 = 149 + 1 + 31 150 + 31 = 81 * Add and subtract pairs of 1 and 2 digit numbers * Add and subtract 3 digit numbers and 1 digit, 3 digit and tens, 3 digit and hundreds * Recognise fractions and pairs of fractions equivalent to one * Calculate with fractions that have same denominator within one whole e.g 1/7 + 2/7 = 3/7 * Recognise and use inverses with + and -, multiplication and division Use in mental calculations • Partition into '5 and a bit' when adding 6, 7, 8 or 9, then recombine. i.e. 27 + 8 = 27 + 3 + 5 = 35 • Add or subtract mentally a 'near multiple of 10' to or from a two-digit number e.g. 53 + 71 = 58 + 70 + 1 • Identify near doubles e.g. 18 + 16 is double 18 and subtract 2 or double 16 and add 2. Understand doubling as inverse of halving • Use patterns of similar calculations. 15 + 17 = 32 therefore 150 + 170 = 320 • Say or write a subtraction statement corresponding to a given addition statement. e.g. 16 + 13 = 29, 29 – 13 =16 etc • To multiply a number by 10/100, shift its digits one, two places to the left • Use knowledge of number facts and place value to multiply or divide by 2,3,4,5, 10 and 100 Say or write a division statement corresponding to a given multiplication statement	• Find what must be added to any multiple of 100 to make 1000 e.g. 300 + □ = 1000 • Add or subtract any pair of two-digit numbers, with and without crossing a tens boundary or 100 e.g. 33 + 45, 87 - 12 • Find what must be added to/subtracted from any two-digit number to make the next higher/lower multiple of 10, e.g. 64 + □ = 70 56 - □ = 50 • Subtract any 3-digit number from any three-digit number when the difference is less than 10 e.g. 458 - 451 • Find what must be added to /subtracted from any three-digit number to make the next higher/lower multiple of 10, e.g. 647 + □ = 650 • Double any number to at least 20 e.g. double 18 and corresponding halves, ½ 36 Double 60 halve 120 • Multiply single digit numbers by 10 or 100 e.g. 6 x 100 • Divide any multiple of 10 x 10 e.g. 60 ÷ 10

Year 4 end of year Mental Calculations expectations

Rapid Recall and Counting	Children should be able to use the following **Mental Strategies**, as appropriate for mental calculations	Children should be able to **Calculate Mentally**

• Count in multiples of 2,3,4,5,6,7,8,9,10,25,100 ,1000 from any number
• Say 10,100,1000 more or less than a number
• Count up through the next multiple of 10, 100 or 1000 e.g. 789, 799, 809 etc
• Reorder numbers in a calculation
• Count in fractions and decimal fractions forwards and backwards
• Double any two or three digit number
• Halve any 2 or 3 digit number
• Know all multiplication tables to 12x12
• Know division tables up to multiple 12x12
• Know decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and any number of tenths and hundredths
	• Round any number to nearest 10 or 100
	• Use place value to aide calculation
	• Show use of number facts in mental calculation strategies and be able to talk about them
	• Calculate with increasingly complex numbers eg. 12,462 – 2,400 or 12,462 + 600
	• Bridge through a 100 or 1000
	  ○ 89 + 67 (67 = 11 + 56)
	  ○ 89 + 11 + 56 = 156
	• Add or subtract 9,19,29,11, 21 etc by rounding and compensating
	• Use knowledge of near doubles
	• Continue to use the relationship between addition and subtraction, multiplication and division
	• Use knowledge of multiplication and division facts to calculate questions such as 640 divided by 8,
	• Use distributive law to derive facts such as 39 x 8 = 30x8 + 9x 8
	• Double two and three digit numbers
	• Partition to carry out multiplication
	• 56 x 7 =
	• (50 x 7) + (6 x 7) =
	• 350 + 42 - 392
	• Use closely related facts to carry out multiplication and division
	• e.g. 7 x 6 = 42 therefore 70 x 6 = 420 or
	• 6 x 8 = 6 x 4 x 2 =
		• Find what must be added to any two digit number to make 100 e.g. 37 + ? =
		• Add or subtract any pair of two-digit numbers e.g. 38 + 85, 92 - 47
		• Find out what must be added to/subtracted from any two or three digit number to make the next higher/lower multiple of 100
		  ○ e.g. 374 + ? = 400, 826 - ? = 800
		• Subtract any four-digit number from any four-digit number when the difference is small
		• e.g. 3641 – 3628 or 6002 - 5991
		• Double any whole number from 1 to 50 e.g. double 36 and find all the corresponding halves e.g. 96 ÷ 2
		• Double any multiple of 10 to 500 e.g. 380 x 2 and find all the corresponding halves e.g. 760 ÷ 2, 130 ÷ 2
		• Double any multiple of 5 to 100 e.g. 65 x 2
		• Multiply any two-digit number by 10 e.g. 26 x 10
		• Divide a multiple of 100 by 10 e.g. 600 ÷ 10
		• Multiply any two-digit multiple of 10 by 2, 3, 4, 5 e.g. 60 x 4, 80 x 3

Year 5 end of year Mental Calculations expectations

Rapid Recall and Counting	Children should be able to use the following **Mental Strategies**, as appropriate for mental calculations	Children should be able to **Calculate Mentally**
• Count forwards and backwards in steps of 10,100,1000 or 10,000 for any given number up to 1,000,000 • Round numbers up to 1,000,000 to nearest 10,100,1000,10,000 and 100,000 • Count in decimal fractions and decimals understanding the place value of each digit • Count in fractions and recognise equivalents • Reorder numbers in a calculation • Double any number with up to 2 decimal places • Halve any number with up to 2 decimal places • Recall quickly multiplication and division facts up to 12x12 and use them to multiply and divide pairs of multiples of 10 and 100 • e.g. 30 x 70, 240 ÷ 40 = 60 • Identify pairs of factors for 2 digit whole numbers • Recall prime numbers to 19 • Know connections between percentages, fractions and decimals	• Use estimation in calculating and verbalise • Calculate whether a number up to 100 is prime • Add and subtract, multiply and divide mentally with increasingly large numbers, practicing speed and fluency • Use partitioning and place value in calculation • Add or subtract the nearest multiple of 10, 100 or 1000 then adjust • Use doubling and halving • Identify near doubles and use in strategies to calculate • Use factors e.g. 15 X 6 15 X 3 = 45 45 X 2 = 90 • Work out sixths by halving thirds etc. • Use closely related facts to carry out multiplication and division 11 x 15 = (8 x 15) + (2 x 15) + (1 X 15) (8 = 2 x 2 x 2) therefore 15 x 2 x 2 x 2 = 120 120 + 30 + 15 = 165 • Use the relationship between addition and subtraction, multiplication and division	• Add or subtract any pair of three-digit numbers e.g. 560 + 250, 620 - 380 • Find what must be added to a decimal fraction with units and tenths to make the next higher whole number e.g. 4.3 + ? = 5 • Add or subtract any pair of decimal fractions each with units or tenths, or each with tenths and hundredths e.g. 5.7 + 2.5, 0.63 – 0.48 • Subtract a four digit number just less than a multiple of 1000 from a four-digit number just more than a multiple of 1000 e.g. 5001 - 1997 • Multiply any two or three-digit number by 10 or 100, e.g. 26 x 10, 79 x 100 • Divide a multiple of 100 by 10 by a single-digit e.g. 60 x 7 • Find 50%, 25% 10% of a small whole number or quantities e.g. 25% of £8 • Calculate complements of 1 with two decimal numbers to two places

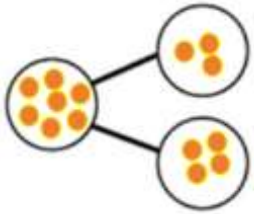
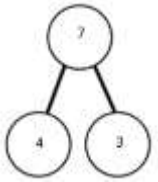
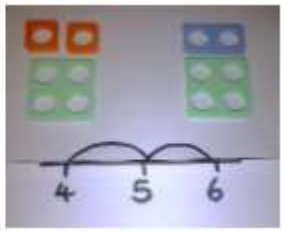
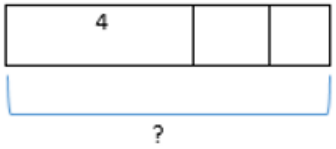
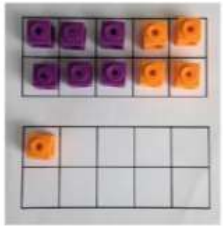
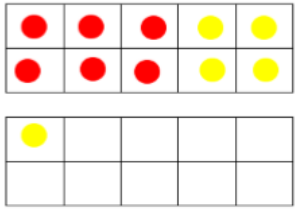
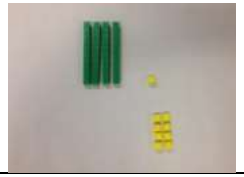
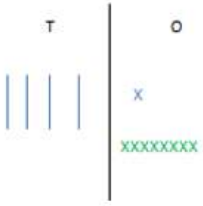
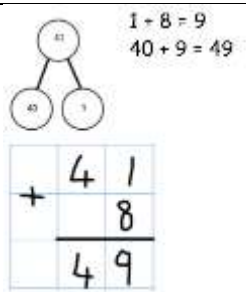
Year 6 end of year Mental Calculations expectations

Children should be able **to Rapidly recall**	Children should be able to use the following **Mental Strategies,** as appropriate for mental calculations	Children should be able to **Calculate Mentally**
• Be able to order to 10 million • Continue to count regularly, whole numbers, fractions, decimals, negative numbers • Introduce counting in binary numbers • Generate linear number sequences including negative and decimal numbers e.g. 1.4, 1.1, 0.8 • Know by heart all the squares and square roots of numbers between 12 x 12 • Recognise and recall factors of numbers up to 100 and corresponding multiples of 100 • Use knowledge of place value and x facts to 10- x 10 to derive related x / ÷ facts • (e.g. 0.8 x 7 = 5.6) • Know by heart test of divisibility for multiples of 2,3,4,5,6,9 10 and 12	• Consolidate all strategies from previous years • Use known number facts and place value to add or subtract pairs of three digit multiples of 10 and two digit numbers with two decimal places • Add or subtract the nearest multiple of 10 or 100, 1000 10,000, then adjust • Continue to use the relationship between addition and subtraction, multiplication and division • Use factors e.g. 35 x 18 • 35 x 2 x 3 x 3 • 70 x 3 x 3 • 210 x 3 = 630 • Use knowledge of place value and number bonds to aide calculation • Use doubling and halving • Use closely related facts to carry out multiplication and division. • Work out 17 times table by adding 7 and 10 times facts and other multiplication tables • Use the relationship between multiplication and division e.g. 0.75 x 4 = 3 buy one get three free • Calculate with unit fractions and use the knowledge of this to see inverse. $\frac{1}{4}$ of a length is 36 so the total length is 36 x 4 = 144. • Use knowledge of fractions and decimals to calculate remainders	• Practice mental calculations and ensure an increased speed of complex calculations • Perform mental calculations with mixed operations • • Multiply any two-digit number by a single-digit e.g. 34 x 6 • Multiply any two-digit number by 50 (multiply by 100 and halve answer) e.g. 23 x 50 or by 25 (multiply by 100 and divide by 4) e.g. 47 x 25 • Multiply or divide any whole number by 10 or 100 giving any remainder as a decimal ○ e.g. 47 ÷ 10 = 4.7, 1763 ÷ 100 = 17.63 • Find squares of multiples of 10 to 100 • Find any multiple of 10% of a whole number or quantity e.g. 70% of £20, of 5 kg of 2 metres

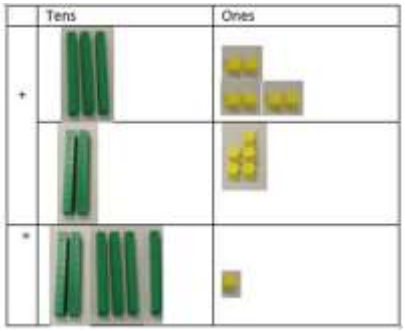
Calculation guidance by year group						
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole:part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method no regrouping.	Column method regrouping. (up to 3 digits)	Column method regrouping. (up to 4 digits)	Column method regrouping. (with more than 4 digits) (Decimals with the same amount of decimal places)	Column method regrouping. (Decimals - with different amounts of decimal places)
Subtraction	Taking away ones. Counting back. Find the difference. Part whole model. Make 10.	Counting back. Find the difference. Part whole model. Make 10. Column method - no regrouping.	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals - with the same amount of decimal places)	Column method with regrouping. (Decimals - with different amounts of decimal places)
Multiplication	Doubling. Counting in multiples. Arrays (with support)	Doubling. Counting in multiples. Repeated addition Arrays - showing commutative multiplication	Counting in Multiples. Repeated addition. Arrays - showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping. Division within arrays.	Division within Arrays Division with a Remainder Short division (2 digits by 1 digit - concrete and pictorial)	Division within arrays. Division with a Remainder. Short division (up to 3 digits by 1 digit - concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2-digit number - interpret remainders as whole numbers, fractions or round)

Addition

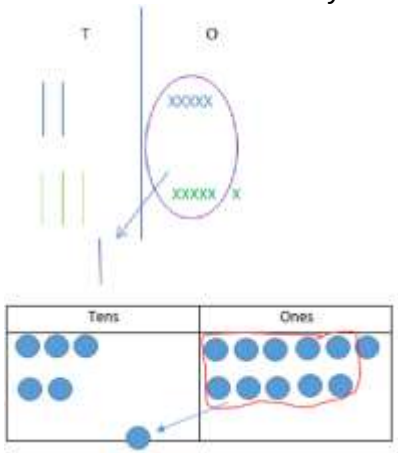
Key Language
Sum, total, parts and wholes, plus, add, altogether, more than, is equal to, is the same as

Concrete	Pictorial	Abstract
Combining two parts to make a whole (use a range of resources e.g. eggs, shells, toy dinosaurs.  		4 + 3 = 7 (four is a part, 3 is a part and the whole is seven) 
Counting on using a number line by using cubes or Numicon  	A bar model which encourages the children to count on 	The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? 4 + 2 
Regrouping to make 10 by using ten frames and counters/cubes or using Numicon:   	Children to draw the ten frames and counters/cubes Use pictures or a number line. Regroup or partition the smaller number to make 10. 	Children to develop an understanding of equality e.g. 6 + ☐ = 11 6 + 5 = 5 + ☐ 6 + 5 = ☐ + 4
TO + O using base 10 Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the concrete using a symbol e.g. lines for tens and dots/crosses for ones. 	41 + 8 

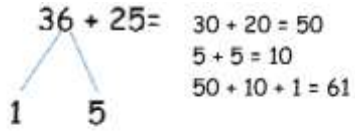
TO + TO using base 10
Continue to develop understanding of partitioning and place value and use this to support addition. Begin with no exchanging.
36 + 25



This could be done one of two ways:



Looking for ways to make 10

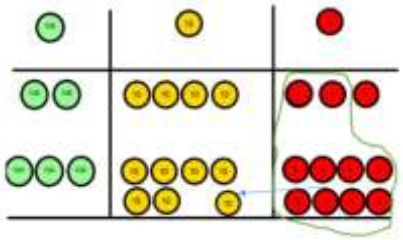


Formal method:

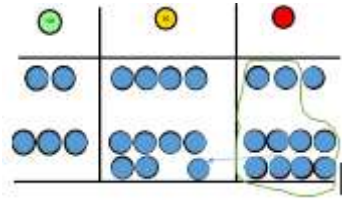
$$\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ \hline 1 \end{array}$$

Use of place value counters to HTO + TO, HTO + HTO etc

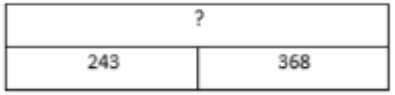
Once the children have had practice with this, they should be able to apply it to larger numbers and abstract



Children to represent the counters, for example, as below

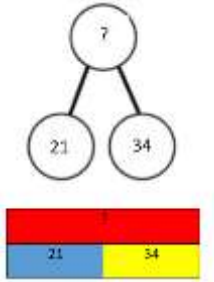


For word problems, draw a bar model to represent the problem



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$$

Fluency variation: different ways to ask the children to solve 21 + 34



Sam saved £21 one week and £34 another. How much did he save in total?

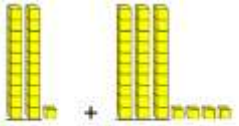
21 + 34 = 55 Prove it!
(a reasoning task but the children need to be fluent at representing it)

$$\begin{array}{r} 21 \\ +34 \\ \hline \quad \end{array}$$

21 + 34 =

☐ = 21 + 34

What's the sum of twenty one and thirty four?



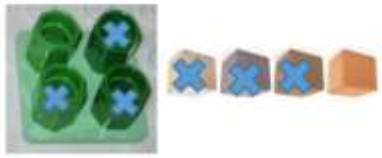
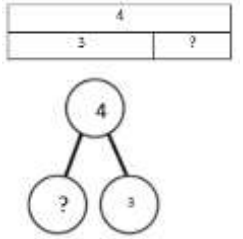
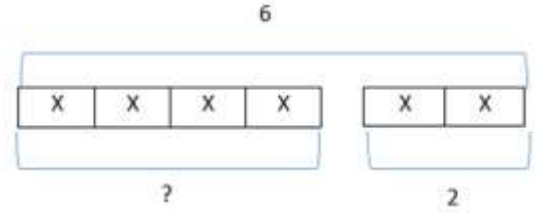
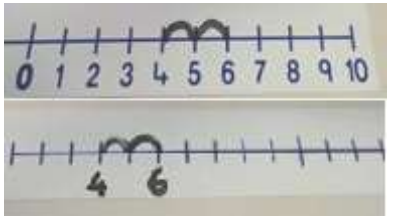
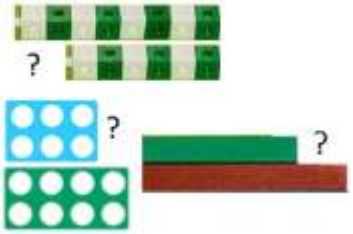
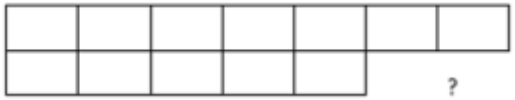
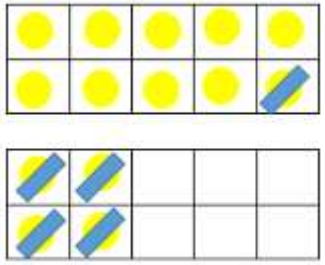
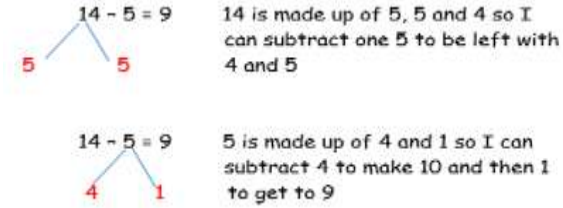
Always use missing digit problems too:

Tens	Ones
○○	●
○○○	?
?	4

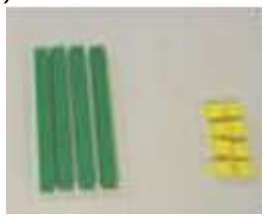
	Step	Augend + Addend = Sum	Examples	Missing number problems
Year R	1	1 digit + 0	5 + 0 =	8 + 2 = □
	2	1 digit + 1 digit total less than 10	4 + 3 =	□ + 2 = 10
	3	1 digit + 1 digit total more than 10	8 + 5 =	8 + □ = 10
	4	1 digit + 1 digit + 1 digit = less than 10	2 + 4 + 1 =	10 = □ + 2
	5	1 digit + 1 digit + 1 digit = more than 10	5 + 3 + 4 =	+ □ + □ = 10
	6	10 + 0	10 + 0 =	10 = □ + □ + □
	7	10 + 1 digit	10 + 7 =	Use 2, 3, 4 digit numbers, as well as decimal numbers up to 3dp when appropriate.
Years 1 and 2	8	Multiple of 10 + 0	40 + 0 =	
	9	Multiple of 10 + 1 digit	70 + 3 =	
	10	Multiple of 10 + Multiple of 10	20 + 50 =	
	11	2 digit + 10	56 + 10 =	
	12	2 digit + 1 digit *(without exchange)*	56 + 3 =	
	13	2 digit + 1 digit *(with exchange)*	38 + 9 =	**Reasoning examples with missing numbers and addition**
	14	2 digit + 1 digit + 1 digit *(without exchange)*	61 + 5 + 2 =	
	15	2 digit + 1 digit + 1 digit *(with exchange)*	78 + 3 + 5 =	
	16	2 digit + 2 digit + 1 digit *(without exchange)*	21 + 45 + 2 =	
	17	2 digit + 2 digit + 1 digit *(with exchange)*	21 + 38 + 4 =	3□ + □5 = 67
	18	2 digit + 2 digit *(without exchange)*	27 + 52 =	
	19	2 digit + 2 digit (with exchange less 100)	56 + 48 =	2□ + 3□ = 59 Find all possibilities
Years 3 and 4	20	2 digit + 2 digit (with exchange over 100)	89 + 43 =	
	21	2 digit + 2 digit + 2 digit (without exchange)	32 + 51 + 15 =	3 + □ = 9 + 4
	22	2 digit + 2 digit + 2 digit (with exchange)	26 + 51 + 39 =	
	23	100 + 0 stressing 0 as a place holder for the ones	100 + 0 =	□ +1475 = 6□24
	24	100 + multiples of 10	100 + 40 =	
	25	100 + 1 digit (less than 110) stressing 0 as a place holder for 10	100 + 7 =	**Column Method and missing numbers**
	26	3 digit + 1 digit (crossing 10s)	273 + 8 =	
	27	100 + 2 digit numbers	100 + 78 =	
	28	3 digit + 2 digit (without exchange)	271 + 36 =	
	29	3 digit + 2 digit (with exchange)	633 + 89 =	
	30	3 digit + multiples of 100	827 + 400 =	42□ +978 1□01
	31	3 digit + 3 digit (without exchange)	620 + 374 =	
	32	3 digit + 3 digit (with exchange)	736 + 197 =	
	33	3 digit + 3 digit + 3 digit (all multiples of 100)	500 + 200 + 100 =	
	34	3 digit + 3 digit + 3 digit (all multiples of 10)	710 + 940 + 620 =	
	35	3 digit + 2 digit + 2 digit (all multiples of 10)	890 + 60 + 80 =	
	36	3 digit + 2 digit + 2 digit	827 + 72 + 15 =	
	37	3 digit + 3 digit + 3 digit	937 + 511 + 827 =	
	38	4 digits + 4 digits	8374 + 2034 =	62□ +349 976
Years 5 and 6	39	2 digit (1dp) + 2 digit (1dp) (without exchange)	3.5 + 2. 4 =	
	40	2 digit (1dp) + 2 digit (1dp) (with exchange)	8.4 + 1.9 =	
	41	3 digit (1dp) + 2 digit (1dp) (without exchange)	73.4 + 5.5 =	
	42	3 digit (1dp) + 2 digit (1dp) (without exchange)	90.5 + 8.	
	43	Add two numbers of any length, each to 2dp max (include money)	827.81 + 9.21 =	
	44	Add two numbers of any length, each to 3dp max (include money)	82.917 + 6352.3 =	
	45	Addition of three or more numbers of any length up to 3dp	8.287 + 2.27 + 7338.2 =	
Missing number addition questions as appropriate throughout				

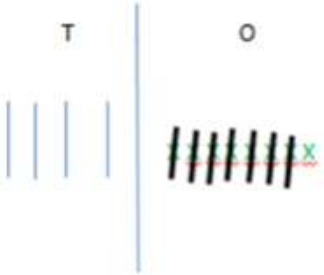
Subtraction

Key Language
take away, less than, the difference, subtract, minus, fewer, decrease, '7 take away 3, the difference is four'

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (use various objects too) rather than crossing out- children will physically remove the object. 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out.  Use of the bar model: 	4 – 3 = □ = 4 – 3 
Counting back (using number lines or number tracks) 	Children to represent what they see pictorially e.g. 	
Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used) 	Children to draw the cubes/other concrete objects which they have used XXXXXXXX XXXXXX Use of the bar model 	Find the difference between 8 and 6. 8 – 6, the difference is? Children to also explore why 9 - 7 = 8 – 6 (the difference, of each digit, has changed by 1 so the difference is the same)
Making 10 (using Numicon or ten frames) 14 – 5  Children could also do this by subtracting a 5 from the 10. 	Children to present the ten frame pictorially 	14 – 5 = 9 You also want children to see related facts e.g. 14 – 9 = 5 Children to represent how they have solved it e.g. 

Column method
(using base 10) 48-7

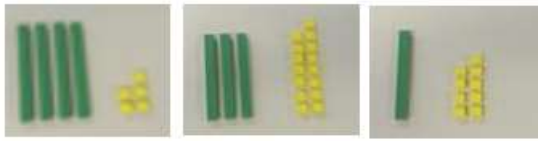




48 - 7 =

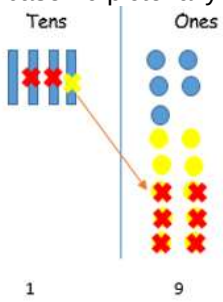
	4	8
-		7
	4	1

Column method (using base 10 and having to exchange)
45 – 26



1) Start by partitioning 45
2) Exchange one ten for ten more ones
3) Subtract the ones, then the tens

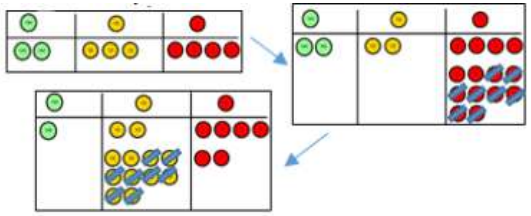
Represent the base 10 pictorially



It's crucial that the children understand that when they have exchanged the 10 they still have 45. 45 = 30 + 15
Column

	~~3~~ 4	15
-	2	6
	1	9

Column method
(using place value counters)
234 - 88

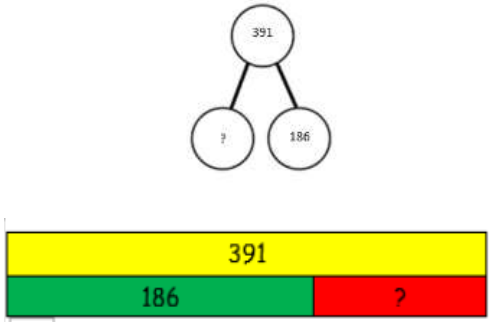


Once the children have had practice with the concrete, they should be able to apply it to any subtraction.

Like the other pictorial representations, children to represent the counters.

$$\begin{array}{r} {}^{2}\cancel{2}^{1}34 \\ -\ 88 \\ \hline 6 \end{array}$$

Fluency variation: different ways to ask the children to solve 391 - 186



Raj spent £391, Timmy spent £186.
How much more did Raj spend?
I had 391 metres to run.
After 186 I stopped. How many metres do I have left to run?

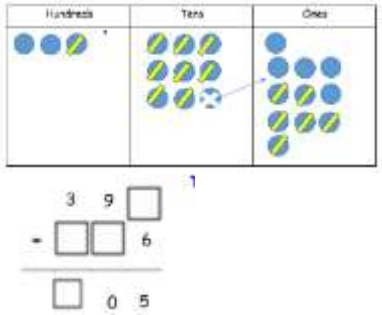
391 – 186

□ = 391 – 186

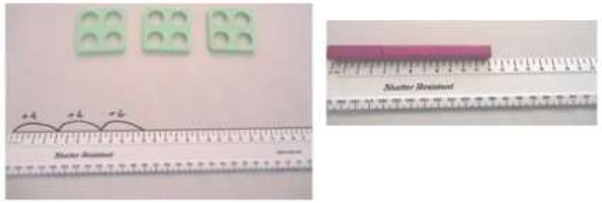
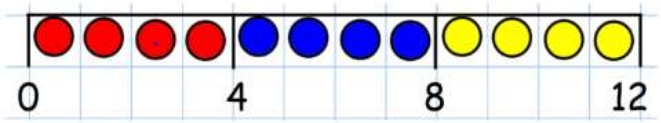
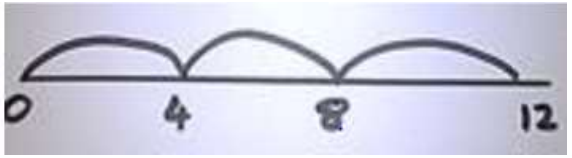
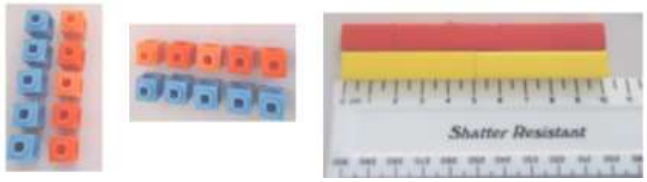
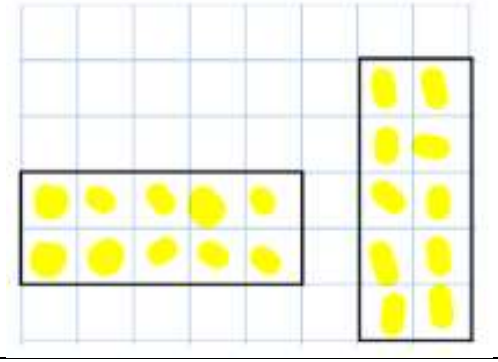
$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

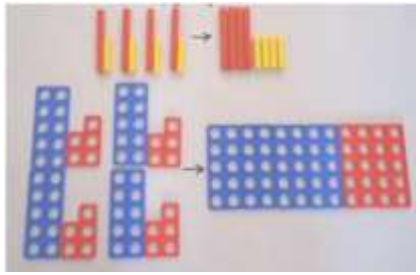
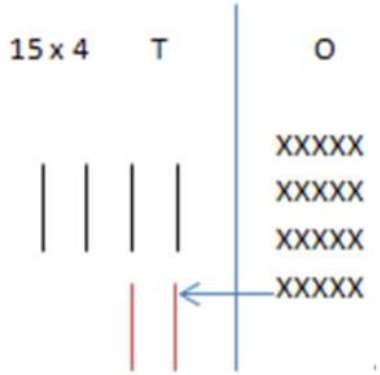
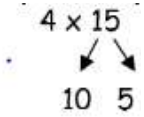
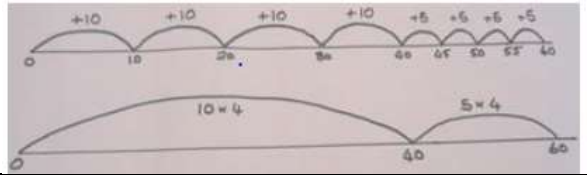
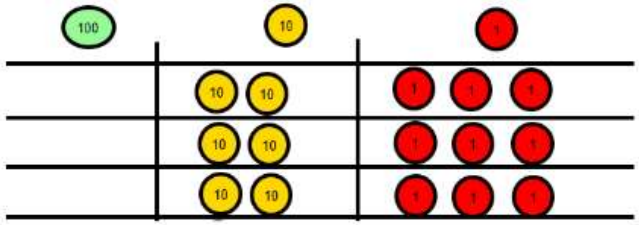
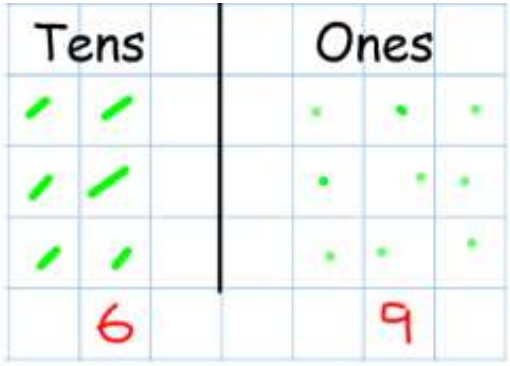
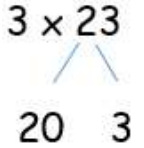
Find the difference between 391 and 186
Subtract 186 from 391.
What is 186 less than 391?

What's the calculation?
What's the answer?

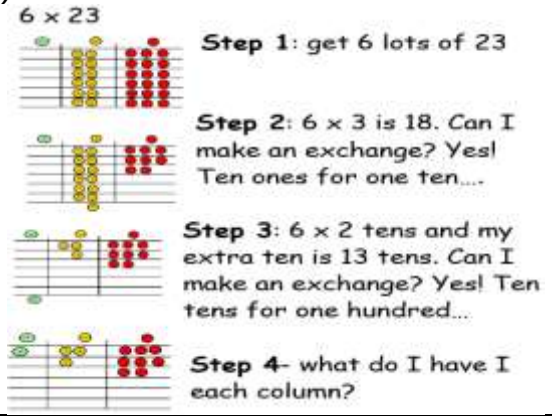


	Step	Minuend – Subtrahend = Difference	Example	Missing number problems	
Year R	1	1 digit - 0 =	7 – 0 =	8 - 2 = □	Use 2, 3, 4 digit numbers, as well as decimal numbers up to 3dp when appropriate.
	2	1 digit - 1 digit (not into negative)	9 – 4 =		
	3	10 - 0	10 – 0 =	8 - □ = 6	
	4	10 - 1 digit	10 – 5 =		
	5	20 – 1 digit	20 – 8 =	□ - 2 = 6	
Years 1 and 2	6	Multiple of 10 - 0	70 – 0 =		
	7	Multiple of 10 - 1 digit	30 – 9 =	6 = □ - 2	
	8	Multiple of 10 - Multiple of 10 (not into negative)	60 – 20 =		
	9	2 digit – 10 (not into negative)	46 – 10 =	□ + □ + □ = 10	
	10	2 digit - 1 digit (without exchange)	89 – 7 =		
	11	2 digit - 1 digit (with exchange)	72 – 8 =	10 - 4 - 3 = □	
	12	2 digit - 2 digit (without exchange)	93 – 51 =		
	13	2 digit - 2 digit (with exchange)	87 – 59 =	**Reasoning examples with missing numbers and subtraction**	
	14	100 - 0 stressing 0 as a place holder	100 – 0 =		
	15	100 - multiples of 10	100 – 60 =	7□ - 3□ = 46 Find all possibilities	
	16	100 - 1 digit	100 – 7 =		
Years 3 and 4	17	3 digit - 1 digit (crossing 10s)	474 – 9 =	Insert the missing symbols (- and =) 19 □ 35 □ 16	
	18	100 - 2 digit numbers	100 – 78 =		
	19	3 digit - 2 digit (without exchange)	836– 325 =	17 - □ = 12 - 4	
	20	3 digit - 2 digit (with exchange)	619 – 445 =		
	21	3 digit - multiples of 100 (no negatives)	276– 200 =	▭ - 666 = 8□5	
	22	3 digit - 3 digit (without exchange)	937 – 522 =		
	23	3 digit - 3 digit (with exchange)	758 – 349 =	**Column Method and missing numbers**	
	24	4 digits - 4 digits	5421 – 6548 =		
Years 5 and 6	25	3 digit (1dp) – 2/3 digit (1dp)	62.5 – 2.9 =	879 -486 □93	
	26	3 digit – 2/3 digit (1dp)	273 – 8.4 =		
	27	3 digit (2dp) - 2/3 digit (1dp)	78.92 – 38.4		
	28	3 digit (1dp) - 2/3 digit (2dp)	73.1 – 29.18 =		
	29	4/5 digit (3dp) – 2/3 digit (1dp)	8.872 – 63.1 =	□72 -356 21□	
	30	3 digit – 4/5 digit (3dp)	287 – 6.547 =		
	31	4 digit (up to 3dp) – 2/3/4 digit (up to 3dp)	91.31 – 9.267 =		
	32	Repeat numbers 25 – 30 with numbers containing zeros	702.04 – 78.9 =		
Missing number subtraction questions as appropriate throughout					

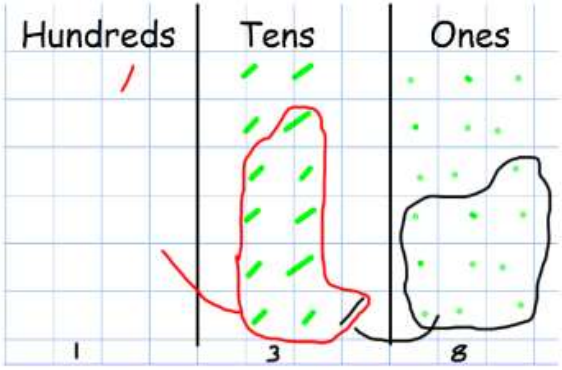
Multiplication	Key Language double, times, multiplied by, product of, groups of, lots of, repeated addition	
Concrete	**Pictorial**	**Abstract**
Repeated grouping/repeated addition (does not have to be restricted to cubes) 3 x 4 or 3 lots of 4 	Children to represent the practical resources in a picture e.g. XX XX XX XX XX XX Use of a bar model for a more structured method 	3 x 4 4 + 4 + 4
Use number lines to show repeated groups 3 x 4 	Represent this pictorially alongside a number line e.g. 	Abstract number line 3 x 4 = 12 
Use arrays to illustrate commutativity (counters and other objects can also be used) 2 x 5 = 5 x 2 	Children to draw the arrays 	Children to be able to use an array to write a range of calculations e.g. 2 x 5 = 10 5 x 2 = 10 2 + 2 + 2 + 2 + 2 = 10 5 + 5 =10

Partition to multiply (use Numicon, base 10, Cuisenaire rods) 4 x 15 	Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like: 	Children to be encouraged to show the steps they have taken  10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used 
Formal column method with place value counters or base 10 (at the first stage - no exchanging) 3 x 23 Make 23, 3 times. See how many ones, then how many tens 	Children to represent the counters in a pictorial way 	Children to record what it is they are doing to show understanding  3 x 20 = 60 3 x 3 = 9 60 + 9= 69 23 x 3 69

Formal column method with place value counters (children need this stage, initially, to understand how the column method works)



Children to represent the counters/base 10,
pictorially e.g. the image below.



The aim is to get to the formal method but the children need to understand how it works.

$$6 \times 23 =$$

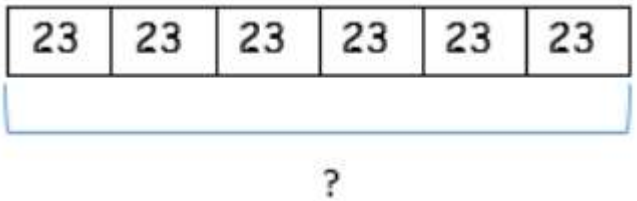
$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

When children start to multiply 3d x 3d and 4d x 2d etc, they should be confident with the abstract:
To get 744 children have solved 6 x 124
To get 2480 they have solved 20 x 124

$$\begin{array}{r} 1\ 2\ 4 \\ \times\ \ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ 1 \end{array}$$

Answer: 3224

Fluency variation, different ways to ask children to solve 6 x 23

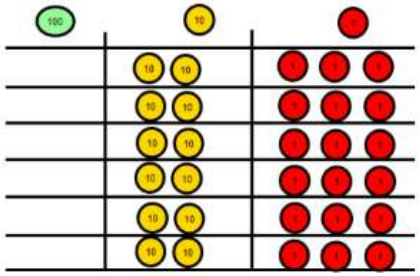


With the counters, prove that 6 x 23 = 138
Why is 6 x 23 = 23 x 6?

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

Tom saved 23p three days a week. How much did he save in two weeks?

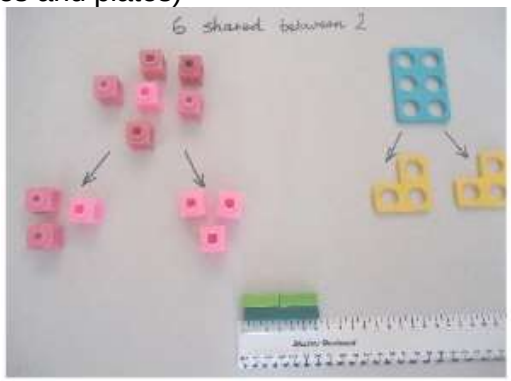
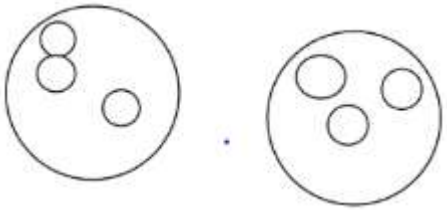
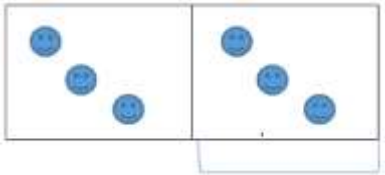
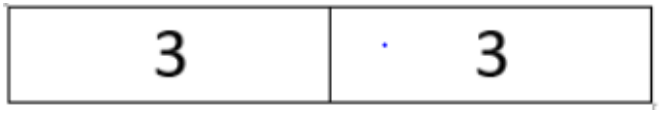
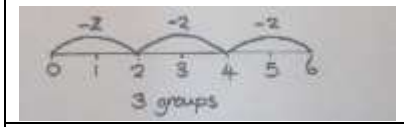
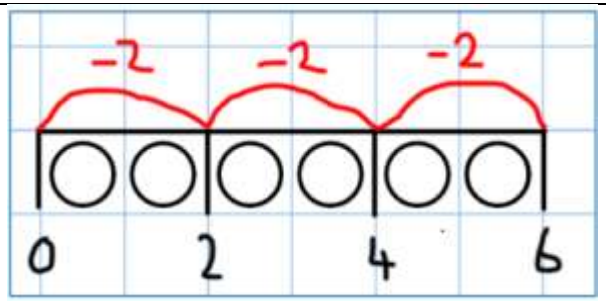
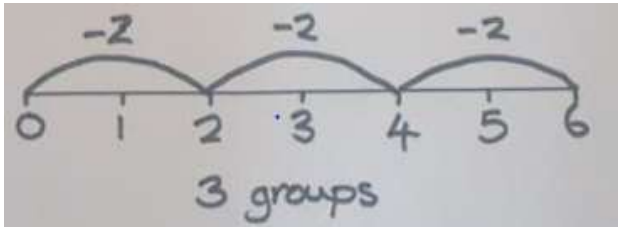
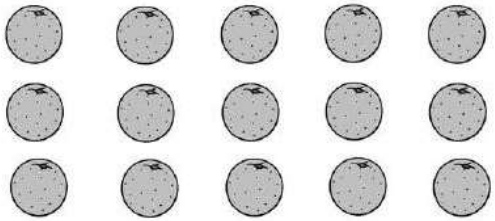
What's the calculation?
What's the answer?

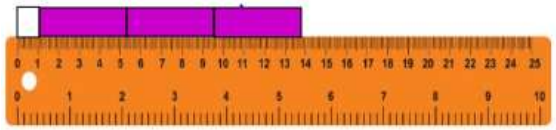
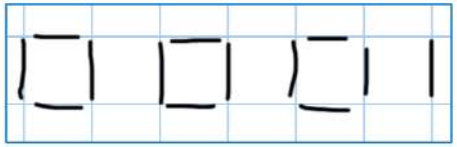
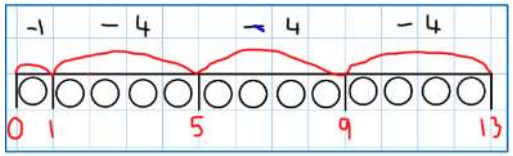
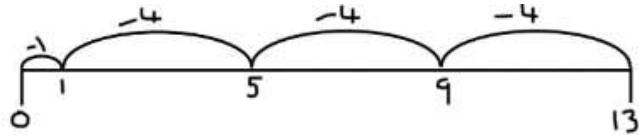
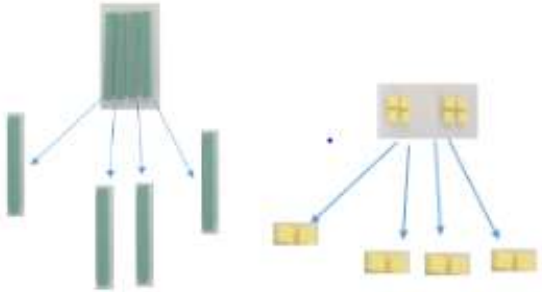
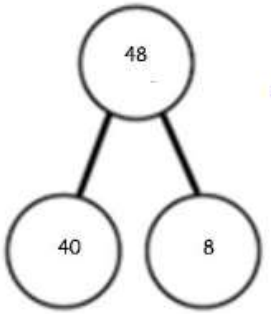
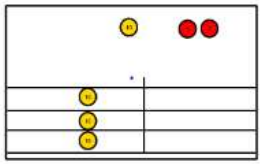
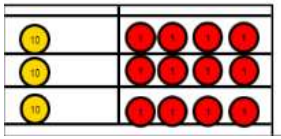
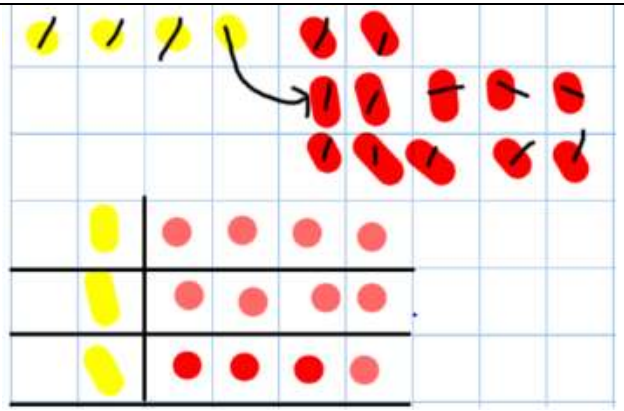


	Step	Factor x Factor = Product	Example	Mixed number problems
YR	1	Introduction to times tables through repeated addition	2 + 2 + 2 =	3 x 4 = □
Years 1 & 2	2	1 digit x 0	7 x 0 = 0	□x 4 =12
	3	1 digit x 1 digit (focused on year times tables)	4 x 3 =	3 x □ = 12
Years 3 and 4	4	2 digit x by 10	24 x 10 =	12 = 3 x □
	5	2 digit x 1 (focused on year times tables)	81 x 1 =	12 = □ x 4
	6	3 digit x 10	56 x 20 =	□ x □x □ = 48
	7	2 digit x 100	546 x 10 =	**Reasoning examples with missing numbers and multiplication**
	8	3 digit x 100	34 x 100 =	
	9	2 digit x multiple of 10	283 x 100 =	
	10	3 digit x multiple of 10	837 x 40 =	24 = □ x □ Which numbers could be written in the boxes?
	11	2 digit x 1000	56 x 1000 =	
	12	3 digit x 1000	726 x 1000 =	□□ x □ Using the digits 2,3,4 how close to 100 can you get?
	13	3 digit x 1 digit	827 x 7 =	
Years 5 and 6	14	2 digit x 2 digit	28 x 82 =	6 x 0.9 = □ x 0.03
	15	3 digit x 2 digit	871 x 27 =	**Column method and missing numbers**
	16	3 digit x 3 digit	701 x 726 =	
	17	2 digit (1dp) x 1 digit	7.8 x 9 =	52 X 6 3□2
	18	3 digit (1dp) x 1 digit	92.4 x 8 =	
	19	2 digit (1dp) x 2 digit	3.5 x 64	
	20	2 digit (1dp) x 2 digit	6.7 x 41	2□4 x 5 □170
	21	3 digit (1dp) x 2 digit	62.3 x 39	
	22	Any money amount x 2 digit	£56.76 x 32	
Missing number problems as appropriate throughout				

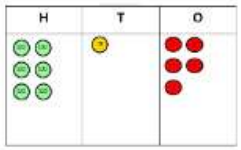
Division

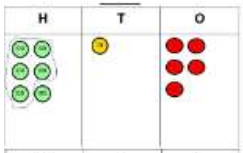
Key Language
share, group, divide, divide by, halve, quarter

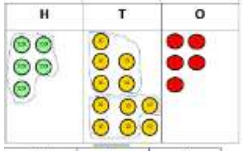
Concrete	Pictorial	Abstract
6 shared between 2 (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates) 	 This can also be done in a bar so all operations have a similar structure. 	6 ÷ 2 = 3 What's the calculation? 
Understand division as repeated grouping and subtracting 6 ÷ 2   		Abstract number line 
Division within arrays  Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

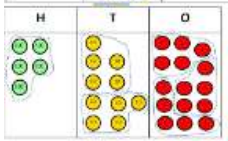
2d ÷ 1d with remainders 13 ÷ 4 = 3 remainder 1  Use of lollipop sticks to form wholes  Use of cuisinere rods and rulers (using repeated subtraction)	Children to have chance to represent the resources they use in a pictorial way e.g. see below:  	13 ÷ 4 = 3 remainder 1 Children to count their times tables facts in their heads 
2d divided by 1d using base 10 (no remainders) SHARING 48 ÷ 4 = 12  Start with the tens.	Children to represent the base 10 and sharing pictorially.	48 ÷ 4  4 tens ÷ 4 = 1 ten 8 ones ÷ 4 = 2 ones 10 + 2 = 12
Sharing using place value counters. 42 ÷ 3= 14  1. Make 42. Share the 4 tens between 3. Can we make an exchange with the extra 10?  Exchange the ten for 10 ones and share out 12 ones		42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

<table>
<tr>
<td>Use of the ‘bus stop’ method using grouping and counters. Key language for grouping – how many groups of X can we make with X hundreds’ – this can also be done using sharing!

615 ÷ 5
 
Step 1: make 615

 
Step 2: Circle your groups of 5

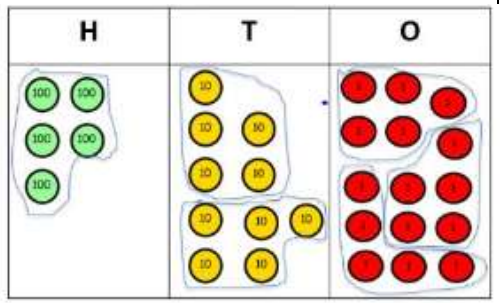
 
Step 3: Exchange 1H for 10T and circle groups of 5

 
Step 4: exchange 1T for 10ones and circles groups of 5</td>
<td>This can easily be represented pictorially, till the children no longer need to do it.
It can also be done to decimal places if you have a remainder.</td>
<td>123
5 ⟌ 6¹1¹5</td>
</tr>
<tr>
<td colspan="4">Fluency variation, different ways to ask children to solve 615 ÷ 5:</td>
</tr>
<tr>
<td>Using the part whole model below, how can you divide 615 by 5 without using the ‘bus stop’ method?</td>
<td>I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group.</td>
<td>5 ⟌ 615

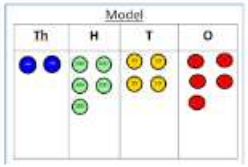
615 ÷ 5 =

☐ = 615 ÷ 5

How many 5’s go into 615?</td>
<td>What’s the calculation? What’s the answer?
 
</td>
</tr>
</table>

Long division

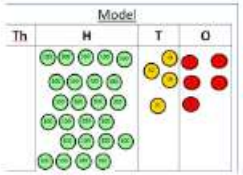
Concrete



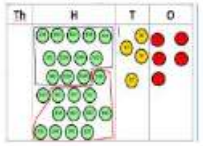
$$\begin{array}{r} 0\,2\,1\,2 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

2544 ÷ 12

How many groups of 12 thousands do we have? None



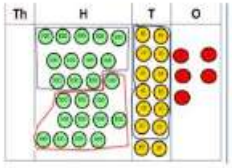
Exchange 2 thousand for 20 hundreds.



$$\begin{array}{r} 0\,2 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$$

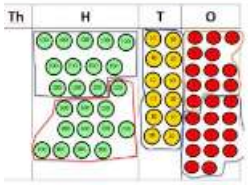
How many groups of 12 are in 25 hundreds? 2 groups. Circle them.

We have grouped 24 hundreds so can take them off and we are left with one.



$$\begin{array}{r} 0\,2\,1 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.



Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2

Pictorial

Children to represent the counters, pictorially and record the subtractions beneath.

Abstract

$$\begin{array}{r} 0 \\ 12\overline{)2544} \end{array}$$

Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.

$$\begin{array}{r} 0\,2 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$$

Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.

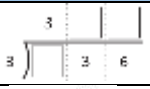
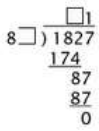
$$\begin{array}{r} 0\,2\,1 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$

Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens?

The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left.

$$\begin{array}{r} 0\,2\,1\,2 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.

	Step	Dividend ÷ Divisor = Quotient	Example	Missing number problems
Years 1 & 2	1	Division as sharing and repeated addition/subtraction (focused on years' timetables)	10 ÷ 2 =	15 ÷ 3 = □
	2	Division with arrays (no remainders)	15 ÷ 3 =	15 ÷ □ = 5
	3	Division with arrays (with remainders)	21 ÷ 4 =	□÷ 3 = 5
Yeas 3 & 4	4	2 digit ÷ 1 digit (no remainders / no exchange)	84 ÷ 4 =	3 = 15 ÷ □
	5	2 digit ÷ 1 digit (no remainders / exchange)	78 ÷ 6 =	5 = □÷ 3
	6	2 digit ÷ 1 digit (remainders / no exchange)	65 ÷ 3 =	**Reasoning examples with missing numbers and division**
	7	2 digit ÷ 1 digit (remainders / exchange)	87 ÷ 5 =	
	8	2 digit ÷ 1 digit (remainders as a fraction of the divisor)	94 ÷ 6 =	
Years 5 and 6	9	3 digit (no zero) ÷ 1 digit (no remainders / no exchange)	936 ÷ 3,	36 ÷ 6 = 12 ÷□
	10	3 digit (no zero) ÷ 1 digit (no remainders / exchange)	959 ÷ 7	Use the inverse to check: 342 x 6 = 18
	11	3 digit (with a 0) ÷ 1 digit (no remainders / no exchange)	602 ÷ 2 =	
	12	3 digit (with a 0) ÷ 1 digit (no remainders / exchange)	504 ÷ 4 =	14□4 ÷ 7 = 222
	13	3 digit ÷ 1 digit (remainders as a fraction of the divisor)	283 ÷ 7 =	3 x 75 = 225 Use this fact to solve 450 divided by 6 =□
	14	3 digit ÷ 1 digit (remainders as a decimal)	823 ÷ 9 =	
	15	4 digit ÷ 1 digit (no remainders)	5904 ÷ 6 =	
	16	4 digit ÷ 1 digit (remainders / exchange)	5631 ÷ 8 =	**Division and missing numbers**
	17	4 digit ÷ 1 digit (remainders as a fraction of the divisor)	9529 ÷ 7 =	
	18	4 digit ÷ 1 digits (remainders as a decimal)	8203 ÷ 6 =	
	19	3 digit ÷ 2 digit (no remainder)	512 ÷ 16	
	20	4 digit ÷ 2 digit (no remainder)	1032 ÷ 12	
	21	3 digit ÷ 2 digit (remainder)	548 ÷ 18	
	22	4 digit ÷ 2 digit (remainder)	1176 ÷ 14	
Missing number problems as appropriate throughout				